Singapore Math
overview

You're probably wondering what's new and different about **Singapore Math**, so here's some "inside information" to help you help your child when s/he has questions…

First, you should know that the **Singapore Math** approach revolves around several key strategies; 1) **thinking about numbers**, 2) **understanding place value,** 3) finding **part-whole** relationships in math, and 4) **breaking down** or **decomposing** numbers into ***friendly*** numbers, ones that are easier to work with in the four **operations** (*addition, subtraction, multiplication, and division)*. Numbers such as **10, 100,** and **1000** are *especially friendly,* so we try as often as possible to work with multiples of these numbers.

Second, **Singapore Math** provides us with logical and often time-saving approaches to math that help us make sense out of problems that may at first seem daunting or tedious. **Mental math** computation is a big component of **Singapore Math** (memorizing number facts to be sure, but also calculating complex problems in our heads). **Model drawing** is an approach to working out word problems where we identify key information in the problem, draw a basic model that incorporates something called a unit bar , and label model with information as we calculate the solution to the problem.

Third, **Singapore Math** teaches students **understanding** of subject matter in stages – beginning with the ***concrete*** (using **manipulatives** such as counters, number disks, place value charts, etc.), then moving to the ***pictorial*** (solving problems where **pictures represent numbers)**, and finally working in the ***abstract*** (where **numbers symbolize values**). Along the way, students learn various ways to work with numbers to help build conceptual understanding, and eventually they master the traditional methods and algorithms.

<u>Number Bonds and Bonding</u>

We start by understanding **number bonds.** They're kind of related to **fact families**, where a number has specific "relatives" in its family. For example **[7,3,4]** is an **addition/subtraction fact family.** You can bond two of the numbers to get the third number (bond 3 and 4 in addition and you get 7; bond 7 and 3 in subtraction and you get 4). It's really helpful to bond numbers that add up to 10 because 10 is an especially ***friendly*** number to work with, so the more you can make 10's out of your numbers during computation, the easier manipulating the numbers becomes (or *multiples* of 10 like 30 or 400 or 8000). Number bonds can also be created with **multiplication (and division) fact families,** using two **factors** and a **product** (for example, **[2,4,8]** or **[3,5,15].**

Branching and Decomposing Numbers

A technique we learn to help us break down or **decompose** whole numbers into parts that are more easily manageable is called **branching.** This just means taking a number, for example, 15 (the **whole**), and **deconstructing** it into 10 and 5 (its **parts**). The lines we draw down from the bigger number to the smaller numbers look like branches. Branching comes in handy when we're adding, say 15 and 24 (the **addends**). We branch 15 onto *friendly* numbers 10 and 5, and we branch 24 into *friendly* numbers 20 and 4. It's easy to add 10 and 20 [30], and **hold onto that number in our head** while we mentally add 5 and 4 together [9], then bring our two answers [30 and 9] together to get 39 as our **sum** or final answer. Here's what using the **branch method** looks like:

Branching Example:

15 + 24 = ?

(10) + 5   4 + (20)

30 + 9 = 39

*(Notice how we **keep the multiples of 10 to the outside of our branching –** in this example, 10 and 20 – <u>and</u> we also **circle them** to highlight how friendly they are!)*

Place Value

In **Singapore Math** we generally work in the Base 10 System. Place value of a number is determined from **right to left**, starting with **ones** and moving along through **tens, hundreds, thousands, ten thousands, one hundred thousands,** to **a million** (and beyond, but in elementary school, we don't usually go past the millions). In class we use tools called **place value charts** to help organize, visualize, and understand what these numbers actually mean. As we perform mathematical operations, we can move **place value disks** (flat, colored chips that are individually labeled by 1's, 10's, 100's, 1,000's, 10,000's, and 100,000's) from column to column on the **place value chart** to demonstrate **renaming** (also called **regrouping, carrying,** and **borrowing)**.

Algorithms

An **algorithm** is a systematic, step-by-step procedure to solve a class of problems such as any of the four operations. For example, in the "traditional" addition and subtraction algorithms, we've learned to line up numbers vertically so that the digits are in the correct place value columns. We've learned to add or subtract the digits *moving from the right column to the left, **regrouping/carrying*** for addition or ***borrowing*** for subtraction if need be, in order to get the correct answer to the problem.

In **Singapore Math**, traditional algorithms such as these are eventually taught and mastered. However, we also teach **alternative algorithms** or approaches to solving problems, often **before** we teach the traditional ones in order to reinforce **understanding of place value** and provide a **variety of problem-solving methods** from which to choose.

Here are some methods of how we teach for understanding, using place value:

Left-to-Right Addition

This algorithm uses something called **expanded notation** where you take a number and *notate it* numerically in a *horizontal* line, *expanding* or *stretching* it to reflect all its place value parts. In **expanded notation**, the number 7,461 would be written as [7,000 + 400 + 60 + 1].

With **left-to-right addition**, we take our numbers and **add the place values** (what each digit represents in value) ***starting on the left and moving to the right*** (greatest values first and least values last; for example, hundreds to tens to ones). Let's say we want to add 723 and 192. First, we want to understand the place value breakdown of the numbers, so we write them out horizontally in expanded notation. Here's an illustration of what the two **addends** in our problem would look like:

723 + 192 = ?

[700 + 20 + 3] and [100 + 90 +2]

Next we add the "like" place values from **left-to-right** (first 100's, then 10's, then 1's), which is really easy because now we're dealing with *friendly* numbers.

700 + 100 = 800
20 + 90 = 110
3 + 2 = 5
?

Vertical Addition Strategy

Another alternative method would be to use a **vertical addition strategy** and work with the numbers lined up vertically (the format we're used to). The difference is, instead of adding columns moving from right to left, we add columns **starting on the *left*** and **moving towards the right** *(in this case, from 100's to 10's to 1's).* We add the **like place values of the digits** (ie. add the hundreds together, then add the tens together, etc.) and line up their sum underneath in a vertical way to solve the problem. *Notice that we're not carrying at all*, but instead, we're recording our numbers as they come up in our addition. This is another alternative method of traditional algorithm of addition.

723
+192
800 *(which is 700 + 100)*
+110 *(which is 20 + 90)*
5 *(which is 3 + 2)*
915

Multiplication Strategies

In **Singapore Math,** we begin teaching multiplication and division concepts (*repeated addition, equal groups,* etc) in **second grade**. We build on what we know about addition, at first setting up and moving around disks on place value charts to help us visualize **groups** of ones and tens, etc. We also use other **manipulatives** such as **counters** (ie. teddy bears, chips, one-inch tiles), to make groups and **arrays** (equal rows and columns) to conceptualize multiplication. By third grade, we're **memorizing times tables** in order to automatically recall multiplication and division facts (an essential time saver when we get into more complex problems!).

We learn early on that **multiplying a number by 10** means you simply append a 0 to the number. Multiplying by 100 means appending two 0's; by 1000, three 0's, etc.

One we get to double-digit (and higher) multiplication (typically fourth grade), we learn some **visually helpful methods to organize numbers** in order to understand what is involved when we multiply. Once we've mastered and understood these *alternative* methods, we learn the **traditional algorithm** (vertical set-up, multiplying right to left, and regrouping/carrying).

Area Model of Multiplication

Using the area model for multiplication is a fast way of multiplying using place value. The model uses a **box design**, where the **factors** (numbers you're multiplying) **are broken down into their place values** (remember *expanded notation*?) and written outside the box (across the top and down the right side). [For example, if you were multiplying 3 and 14, you'd place a 0 and 3 (thick "03" as the number "3") down the right side and put a 10 and 4 in two different spaces above the box.] Top and side numbers on the outside get multiplied, and each product is placed in its appropriate square within the box. Then **all the numbers inside the box are *added up,*** and the **sum** of those numbers is actually the **product** (answer) for the multiplication equation!

Simple Example: **3 x 14 = ?**

10	4	
0	0	0
30	12	3

0 x 4 = 0
0 x 10 = 0 *add*
3 x 4 = 12 *these*
3 x 10 = 30
Total (sum) = 42

More Challenging Example: **35 x 27 = ?**

30	5	
600	100	20
210	35	7

30 x 7 = 210
30 x 20 = 600
5 x 7 = 35
3 x 10 = 100
Total (sum) = 945

35 x 27 = 945 (*the product)*

Lattice Model for Multiplicaion

This algorithm can be an effective tool for learning complicated multi-digit multiplication as it also uses a visually helpful "box organizer." Students draw a **sectioned box** with a **diagonal lattice design**, set up the **factors** (numbers being multiplied) horizontally and vertically along the outside of the box (above the top line and to the right of the vertical right line), multiply vertical and horizontal digits and place "partial" products inside the split windows, then **add the lattice columns along their diagonals** to get the **product** (answer) which is read across the bottom outside the box.

FOIL Method of Multiplication

This horizontal model assigns an order in which to multiply the **place value terms** of the two **factors** in a multiplication problem. **FOIL** is an acronym for remembering the order **–** **F**: multiply **FIRST** term; **O**: multiply **OUTSIDE** terms; **I**: multiply **INNER** terms; and **L:** multiply **LAST** terms. Once you've multiplied all the terms, just add them all together to get your **product.**

	F*irst*		**O***utside*		**I***nner*		**L***ast*	
35 x 27 =	(30 x 20)	+	(30 x 7)	+	(5 x 20)	+	(7 x 5)	= ?
35 x 27 =	600	+	210	+	100	+	35	= 945

Distributive Property for Multiplication Problem_____7 x 27 = ?

The **distributive property** services as our "key" to **rearranging** numbers to create a friendlier order in which to compute. The **distributive property** enables us to **decompose** and **bond** numbers so we can move them around, combine different parts of the whole, look for friendly numbers with which to work, and ultimately come up with the correct solution.

7 x 27 = 7(20 + 7) = (7 x 20) + (7 x 7) = 140 + 49 = **189**

Distributive Property for a Division Problem_________52 ÷ 4 = ?

You've got a **divisor** of 4, so you look for multiples you can **branch** within the **dividend** (52) that could easily be divided by 4 (for example 40 and 12). After branching 52 into 40 and 12, use the **distributive property** to divide both 40 and 12 by 4 to get 10 and 3. Add 10 and 3 to get 13, which is your **quotient** (answer). (To check your work, multiply 13 x 4 and get 52).

52 ÷ 4 = ?

52
/ \
40 + 12
(40 ÷ 4) (12 ÷ 4)
10 + 3
13

52 ÷ 4 = 13
Check work: **13 x 4 = 52**

(Since we're talking "properties" here, we'd just like to mention the **commutative property**, which is the one where you can reverse the order of the numbers you're adding (**addends**) or multiplying **(factors)** and still come up with the same **sum** or **product**. (It's as if the numbers are "commuting" back and forth in both directions.) For example [7 + 8] and [8 + 7] both equal 15; likewise, [5 x 4] and [4 x 5] both equal 20.

Division Strategies

Bonding and **branching** come in handy when we're learning division. **Branching numbers** into **bonds** to **decompose** them into friendlier, more manageable numbers reinforces our understanding of the **part-whole** relationship and allows us to work with simpler numbers. Using **place value charts** to trade disks ("distribute" them) from place value column to place value column helps us parcel out a greater quantity (**dividend**) into a specific number of equal lesser groups (**divisor**) in order to get an answer (**quotient**). (After dividing into **equal groups,** we sometimes end up with some "extras" or a **remainder**.)

Partial Quotient Method of Division

The **partial quotient method** is similar to the traditional "long division" algorithm, but it has the advantage that for some students, it is easier to initially learn and apply. Think of it as a **series of successive approximations or estimates.** (One strategy would be to use multiples of 10's, 100's, and 1,000's as your estimates because they're so easy to divide and multiply, which we'll need to do a lot of as we work the problem.) The **quotient** (answer) is built through **vertical steps** (which resemble the game, "Hangman" a little), and we don't have to get the partial quotient exactly right at each step. [We can just pick *any* number that we think answers the question, **"How many (divisor #)'s can we pull out of (dividend #)?]** We just continue finding **partial quotients** and divide, multiply, and subtract (as we would normally do in long division), until we have no **remainder** or a remainder that is less than the divisor. We then **add the partial quotients** to arrive at the **final quotient.**

Partial Quotient Division Example: **158 ÷ 12 = ?**

	13 r2		
12	158	10	Partial quotient (How many 12's can you pull out of 158? 10?
	120		Multiply 10 x 12 to get 120: then subtract (158 - 120 = 38)
	38	+3	Partial quotient (How many 12's can you pull out of 38? 3?)
	36	___	Multiply 3 x 12 to get 36; then subtract (38 – 36 = 2)
	2	13	How many 12's can you pull out of 2? None! "*Dividend*" of 2 is less than the *divisor* (12), so dividing ends here, and the 2 becomes the *remainder*.
			Now **add all the partial quotients** (10 + 3 = 13).
			Don't forget the **remainder** in your answer!
			Final quotient (*answer*) is 13 r2 **158 ÷ 12 = 13 r2**

Another Partial Quotient Division Example: **7891 ÷ 36 = ?**

	219 r7		
36	7891	100	
	3600		
	4291	100	
	3600	+	
	691	10	
	360		
	331	9	9 X 36 = (9 X 30) + (9 X 6) = [270 + 54] = (270 + 30) + 24 = (300 + 24] = 324
	324	219	Sum of partial quotients
	7		Remainder of 7

Final quotient (*answer*) is 219 r7 **7891 ÷ 36 = 219 r7**

Long Division

After students understand conceptually what's involved when dividing, they can use the **long division algorithm** the "traditional" way – dividing the **dividend** by the **divisor** from left to right, recording the answer above the line, multiplying the answer times the divisor and recording it vertically, subtract, and continuing the pattern of calculation until we get the **quotient** and possibly a **remainder.**

Short Division

Short division can be a tool for students who've mastered **partial quotient** and **long division**. It offers a shortcut to solving long division problems. Basically, you set up the problem as you would long division, but instead of having a long vertical trail of numbers that you've multiplied and subtracted, you take a shortcut and do your division calculations in your head, using **abbreviated notation** to "carry over" or "regroup" any remainders that need to be added to your next number in the dividend. Here's an example:

153 ÷ 2 = ?

	76 r1
2	153

How many 2's can you pull out of 15? (7 with a remainder of 1; place the 1 just before the 3)
How many 2's can you pull out of 13? (6 with a remainder of 1) *Answer*: **76 r1**

Mental Math

The cornerstone of **Singapore Math** is its emphasis on **mental math**, which encourages flexibility and speed when working with numbers. We practice mental math exercises daily in class and learn specific strategies to help us mentally manipulate numbers and solve problems quickly in our head.

Estimation and Rounding

In mental math and in more complicated problem solving, we sometimes challenge students to "sidestep" the precise calculation of a problem and **estimate** the answer instead, sometimes using **rounding** (to 10's, 100's, 1,000's etc.) We round **up** (5's, 50's, 500's or higher, etc.) or round **down** (4's, 40's, 400's or lower, etc.) using **place value. Rounding** is a useful tool that can help make computation easier or give us an early indication of approximately what the answer to a problem will be. It can also help us check our work to see if we're more likely close to the right answer or way off base.

Compensation

Compensation is a technique we use all the time in math without necessarily thinking about it. We use it to convert a problem to a more manageable one in order to calculate the answer more easily. For example, let's say we want to add 99 and 34. 99 is not nearly as easy a number to work with as 100, so by taking 1 away from 34 and adding it to 99, we come up with a *friendlier* number (100), which we find easier to work with to calculate the answer: [100 + 33 = 133]. Another example, this one in multiplication, would be with the problem (50 x 12). We can double the 50 to make it a *friendly* 100, and **compensate** by halving the 12 into [10 and 2] and, using the **distributive property,** multiply (50 x 10) and (50 x 2) to get (500 + 100) which equals 600.

Model Drawing

Remember how daunting all those **word problems** were when you were a math student? **Singapore Math** provides a systematic approach to solving these often puzzling problems – a place to start and an eight-step plan that allows you to set up a visual model to get you through the language and challenge of the problem. Starting in second grade, we learn the sequential steps, which include:

- ***read the entire problem****, first ignoring the actual numbers so we can "visualize" the problem conceptually (****reflective reading****) and then reading it again with all the numbers included*
- ***decide and write down who and/or what*** *the problem is about*
- ***draw unit bars of equal length*** *that we'll eventually adjust as we construct the visual image of the problem.*
- ***reread the problem phrase by phrase and label the unit bars*** *to reflect the information as we reread it*
- ***determine exactly "what" we're being asked to solve, and place a question mark*** *in the place on the model drawing that reflects the "what"*

- ***compute*** *the problem to come up with an answer (show all work!)*
- ***write the answer in a complete sentence*** *that clearly states the solution*

We learned the step-by-step *method* of **how to set up a model drawing** by using "below grade level" problems that we can pretty much figure out in our heads. Once we have a solid foundation of how to draw a model and work our way through it, we take on more challenging word problems where model drawing actually guides us through some really complicated material. As in all approaches we're learning in **Singapore Math**, model drawing is a *strategy* to call on as needed. There are countless strategies we can use when solving mathematical problems, and the more strategies we have in our *toolbox*, the more successful we're likely to be.

That about wraps it up for this draft of ***Singapore Math – Demystified!*** What we're finding as we teach **Singapore Math** is that **math is math**, no matter how you package it. *Number sense, place value, calculation, operations, puzzles, problem solving, numbers, visuals, words, and relationships* – are all familiar parts of math we already use and important elements of strategies we're learning. **Singapore Math** provides us with some new vocabulary and **multiple approaches to solving math problems** – many of them initially more accessible to students than traditional approaches and all of them leading to mathematical mastery.

Thank you for continuing to ask questions, provide feedback, and learn with us in this new and exciting academic adventure that is **Singapore Math!**